Functions (Section 2.3), CS-24, Dr. Ostheimer

Learning objectives:
• to get to know each other and start setting up study partners;
• to practice reading the text;
• to practice using the logical notation taught in CS-14.

You will need your index cards with the definitions of function, one-to-one and onto. You will also need this definition:

**Definition 1.** A relation from set \( A \) to set \( B \) is a subset of \( A \times B \).

Here are 4 sample relations.

- \( A = \{1, 2, 3\}; B = \{5, 6\}; R_1 = \{(1, 5), (1, 6), (2, 5), (3, 5)\}. \)
- \( A = \{1, 2, 3\}; B = \{5, 6\}; R_2 = \{(1, 5), (2, 5), (3, 6)\}. \)
- \( A = \{1, 2, 3\}; B = \{5, 6, 7, 8\}; R_3 = \{(1, 5), (2, 6), (3, 7)\}. \)
- \( A = \{1, 2, 3\}; B = \{5, 6, 7\}; R_4 = \{(1, 5), (1, 7), (2, 5), (3, 6)\}. \)

And here are 4 pretend definitions.

- A relation \( R \) from \( A \) to \( B \) is tall if
  \[ \forall a \in A, \exists b \in B \mid (a, b) \in R. \]

- A relation \( R \) from \( A \) to \( B \) is dark if
  \[ \forall b \in B, \exists a \in A \mid (a, b) \in R. \]

- A relation \( R \) from \( A \) to \( B \) is handsome if
  \[ \forall a_1, a_2 \in A, b \in B, ((a_1, b) \in R \land (a_2, b) \in R) \rightarrow a_1 = a_2. \]

- A relation \( R \) from \( A \) to \( B \) is charismatic if
  \[ \forall a \in A, b_1, b_2 \in B, ((a, b_1) \in R \land (a, b_2) \in R) \rightarrow b_1 = b_2. \]

And here are your questions:

1. Which of the relations \( R_1, R_2, R_3, R_4 \) above is a function?
2. Of those relations that are functions, which are one-to-one?
3. Of those relations that are functions, which are onto?
4. Of those relations that are functions, which are one-to-one correspondences?
5. Of the relations above, which are tall? which are dark? which are handsome? and which are charismatic?
6. In order for a relation \( R \) to be a function, what characteristics must it have? For example, must it be tall, dark and handsome?
7. In order for a relation \( R \) to be a one-to-one function, what characteristics must it have?
8. In order for a relation \( R \) to be an onto function, what characteristics must it have?
9. In order for a relation \( R \) to be a one-to-one correspondence, what characteristics must it have?